Quantized Turbo Equalization for Non-binary LDPC Coded Partial-Response Channels

Nina Zhang, Zhiliang Qin, Yingying Li, Luyan Xing, Qidong Lu, Xiaowei Liu

***Abstract*—In this paper, we consider iterative detection and decoding (i.e., turbo equalization) for nonbinary low-density parity-check (LDPC) coded partial-response channels, where a quantizer is present to discretize the continuous received signal. We propose a turbo equalizer that uses the pre-computed quantized channel transition probabilities in the symbol-level BCJR channel detection algorithm, which significantly reduces the computational complexity by avoiding real-time floating-point multiplications. The proposed approach is further extended to nonbinary LDPC coded bit-patterned media recording (BPMR) channels. Simulation results show that with a small number of quantization bits, the proposed receiver approaches closely the performance of the conventional turbo equalizer operating on unquantized signals.**

***Index Terms*—BCJR algorithm, nonbinary LDPC codes, quantization, turbo equalization**

I. INTRODUCTION

Motivated by the erformance gains provided by low-density parity-check (LDPC) codes over additive white Gaussian noise (AWGN) channels, iterative detection and decoding for partial-response channels, which is named turbo equalization [1], has drawn much interest in recent years. A turbo equalizer is typically composed of an inner soft-input/soft-output (SISO) channel detector and an outer LDPC decoder with extrinsic information exchanged in-between at each iteration. The optimal realization of channel detection is based on the BCJR algorithm [2], which usually assumes that the input to the channel detector is infinitely precise (i.e., unquantized) and hence needs to compute in real time the Euclidean distances between received signals and noiseless channel outputs. This may become computationally intensive for practical implementations of high-speed applications.

In a practical system, a quantizer is always present at the front end and the input to the receiver takes only a fixed number of discrete values. Under this constraint, the conventional BCJR detector is no longer optimal. In [3], a modified parallel turbo decoder based on quantized channel observations was proposed. In [4], several quantization schemes were investigated for binary LDPC decoding. While the previous works in [3] and [4] consider only AWGN channels, the effect of quantization on the performance of turbo equalization over magnetic recording channels has not been addressed so far.

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In this paper, we show that a quantized partial-response channel can be effectively characterized by a set of probabilities corresponding to transitions between noiseless channel outputs and quantized input signals. Based on this observation, we propose a low-complexity symbol-level BCJR detector that is optimal for producing soft information over quantized nonbinary LDPC coded channels. The proposed detector uses pre-computed transition probabilities in the branch metric calculation, thus avoiding the necessity to perform real-time floating-point multiplications and normalizations. Following that, we further extend the proposed approach to a more practical bit-patterned media recording (BPMR) channel model [5]. Simulation results show that with 6-bit uniform quantization, the proposed receiver achieves bit-error-rate (BER) performance that is very close to that of the conventional receiver operating on unquantized signals. Moreover, when severe constraints are imposed on the number of quantization bits, we show that a 5-bit non-uniform quantizer significantly improves performance and reduces the loss to 0.1 dB.

II. SYSTEM MODEL

We consider nonbinary LDPC codes transmitted over an idealized partial-response channel, where the data sequence **d** is converted to a symbol sequence and encoded by a nonbinary LDPC encoder defined over Galois field GF(2^s) [6]. The code symbols **c** are then mapped to binary code bits **b** and transmitted over the channel. At the time index k, k=0,…,Ns−1, the received signal **r** is given by,

$$r_k = \sum_{i=0}^{m} h_i b_{k-i} + w_k \tag{1}$$

where w_k denotes AWGN with variance $\sigma^2=N_o/2$, N_o is the noise one-sided power spectral density, $\mathbf{h}=\{h_i\}$, $i=0,\ldots,m$, denotes channel coefficients, m is the channel memory length, and N is the number of nonbinary LDPC code symbols, respectively. In a practical system, r_k is passed through a quantizer to generate quantized samples r_k^q, where the superscript q denotes quantization.

In [7], it is shown that when applied to nonbinary coded systems, the conventional bit-level BCJR channel detector [2] suffers noticeable performance degradation due to information loss arising from conversions between bit-level log-likelihood ratios (LLR) and symbol-level LLR. The symbol-level BCJR detector, which produces directly nonbinary LDPC code symbol LLR, is optimal in this scenario and provides better performance. Hence, in this paper we focus on the effect of quantization on the symbol-level BCJR detector.

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III. Turbo Equalization with Quantized Input

A. *Quantized Channel Transition Probabilities*

As shown in Fig. 1, a quantized partial-response channel can be described by a set of probabilities $P_{i,j}$ representing transitions between the ith noiseless channel output x_i and the jth quantized value q_j, where i=0,…,M−1, $M=2^{m+1}$ is the number of all possible combinations of m+1 bits with each combination corresponding to a noiseless channel output, j=0,…,L−1, and $L=2^n$ is the number of quantization intervals. The probability $P_{i,j}$ can be obtained as,

$$\begin{aligned} P_{i,j} &= P\left(r^q = q_j \middle| x_i\right) \\ &= P\left(r \in T_j \middle| x_i\right) \\ &= \int_{a_j}^{a_{j+1}} \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(r-x_i)^2}{2\sigma^2} \right) \right) dr \end{aligned} \tag{2}$$

where $T_j=(a_j,\ a_{j+1}]$ denotes the jth quantization interval, $\{a_0,a_1,\ldots,a_L\}$ denotes the set of quantization boundaries with $a_0=-\infty$ and $a_L=\infty$, and $P_{i,j}$ is the conditional probability density function (pdf) of the unquantized signal r belonging to the quantization interval T_j. Note that x_i is not binary but real-valued here when the partial-response channel is considered. The quantization operation leads to a discrete memoryless channel (DMC) with M (M>2) inputs and L outputs. Given the quantizer specifications and the channel pdf, the transition probabilities (2) can be computed in advance and stored in a matrix of size M×L as

$$\mathbf{P} = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,L-1} \\ p_{1,0} & p_{1,1} & \cdots & p_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M-1,0} & p_{M-1,1} & \cdots & p_{M-1,L-1} \end{bmatrix} \tag{3}$$

It is noted that for each signal-to-noise ratio (SNR), a specific transition probability matrix **P** can be obtained.

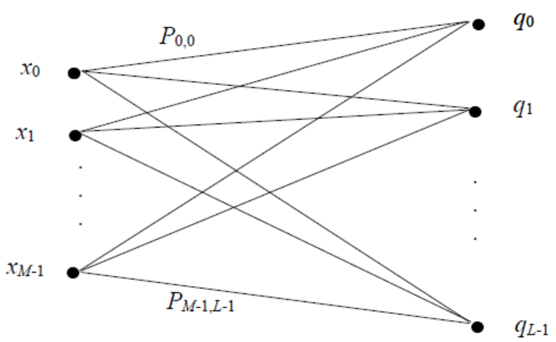

Fig. 1. Representation of a quantized partial-response channel by a transition probability matrix.

B. *Quantized Symbol-Level BCJR Channel Detector*

Assume that a non-binary LDPC code over GF(2^s) is transmitted over the channel. As represented by the factor graph [6] in Fig. 2, we can convert the conventional bit-level channel to a symbol-level dicode channel by grouping s adjacent bits into a symbol. The symbol-based BCJR algorithm operates on the equivalent symbol-level channel trellis to produce directly the *a posteriori* probability (APP) LLR of nonbinary LDPC code symbols **c** as,

$$\begin{aligned} \Lambda(c_k) &= \log \frac{P(c_k = c|\mathbf{r})}{P(c_k = 0|\mathbf{r})} \\ &= \log \frac{\sum_{\mathbf{S}_k \to \mathbf{S}_{k+1}: c_k = c} p(\mathbf{S}_k, \mathbf{S}_{k+1}, \mathbf{r})}{\sum_{\mathbf{S}_k \to \mathbf{S}_{k+1}: c_k = 0} p(\mathbf{S}_k, \mathbf{S}_{k+1}, \mathbf{r})} \end{aligned} \tag{4}$$

where $\mathbf{S}_k$ denotes the state in the kth symbol-level trellis stage, k=0,…,N−1, c denotes a symbol from GF(2^s) over the trellis branch corresponding to the state transition $\mathbf{S}_k \rightarrow \mathbf{S}_{k+1}$, and c=0,…,2^s−1. The choice of the base c_k=0 is arbitrary.

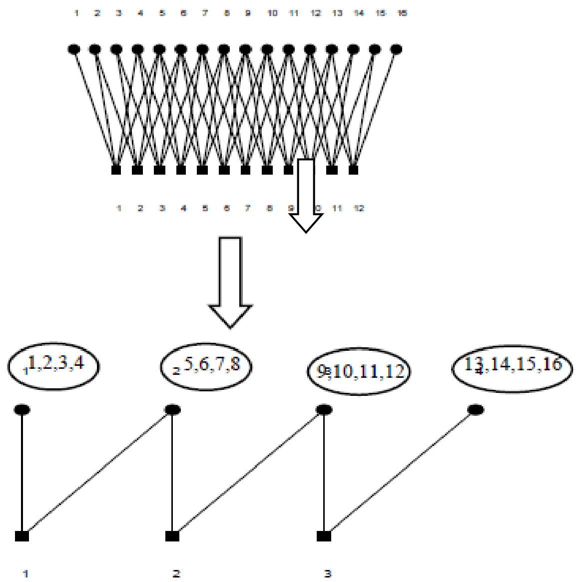

Fig. 2. Conversion of a 5-tap E2PR4 bit-level channel into an equivalent symbol-level dicode channel. Circles represent bits/symbols; while squares represent ISI channel constraints. The nonbinary LDPC code is defined over GF(24).

Similar to the conventional bit-level BCJR algorithm [2], the symbol-level BCJR algorithm performs the forward recursion, the backward recursion and the branch metric calculation over the equivalent trellis to compute α, β, and γ, respectively. Specifically, the branch metric γ in the log-domain can be computed as

$$\gamma(\mathbf{S}_k \rightarrow \mathbf{S}_{k+1}) = -\frac{1}{2\sigma^2} \sum_{l=0}^{s-1} (r_{l+ks} - x_{l+ks})^2 + \lambda(c) \tag{5}$$

where the s-tuple channel observations r and noiseless channel outputs x are labelled over the branch $\mathbf{S}_k \rightarrow \mathbf{S}_{k+1}$, respectively, and λ denotes the *a priori* LLR of nonbinary LDPC code symbols fed back from the decoder in the previous iteration. The noise variance σ^2 is assumed to be known to the receiver. Based on (5), the symbol-level BCJR algorithm operating on unquantized received signals needs to compute in real time Euclidean distances over each trellis branch, which involves s multiplications, s additions, and one normalization operation by the factor $2\sigma^2$, respectively.

When a quantizer is used to discretize the received signal, we assume that the lth noiseless output x_{l+ks} in (5) corresponds to the ith branch in the *bit-level* channel trellis, i=0,…, M−1, $M=2^{m+1}$, and the received signal r_{l+ks} belongs to the jth quantization interval, j=0,…,L−1. Hence, the branch metric over quantized channels is given by

$$\gamma(\mathbf{S}_k \rightarrow \mathbf{S}_{k+1}) = \sum_{l=0}^{s-1} \log\left(P_{i,j}^{(l)}\right) + \lambda(c) \quad (6)$$

where the log-domain transition probabilities $P_{i,j}$ are retrieved from the lookup table as computed in (3). Compared with (5), the number of operations required for quantized channels is reduced to s table lookups and s additions per branch. Moreover, the memory size for storing these tables can be further reduced in practical implementations. Simulation results show that the system performance is insensitive to the lookup table chosen in the branch metric computation.

IV. Performance Results

A. E2PR4 Channels

The nonbinary LDPC code used in the simulation is based on the Netto's construction of the Balanced Incomplete Block Designs (BIBD) [8]. Consider GF(97) with α=3 as a primitive element. The associated BIBD has 97 points and 1552 blocks. These blocks can be grouped into 16 cyclic classes. Taking the first 12 cyclic classes, we form a 97×1164 binary matrix with column weight 3 and row weight 36, respectively. The null space of the matrix defines a rate-0.917 (1164, 1067) binary LDPC code. The corresponding nonbinary code is obtained by substituting 1 entries of the binary matrix randomly with nonzero elements of GF(24). The LDPC decoding is based on the floating-point implementation of the sum-product algorithm [6] over the symbol-level code graph. For the considered channel, the equivalent symbol-level dicode trellis is converted from the bit-level E2PR4 trellis by grouping s=4 adjacent bits into a symbol.

In Fig.3, we present the performance of turbo equalization over the quantized nonbinary LDPC coded ideal E2PR4 channel, whose coefficients are given by h={1,2,0,−2,−1}. As in the optimal iterative schedule [9], we perform the BCJR detection once for each step of LDPC decoding, which itself involves one local iteration between symbol nodes and check nodes. The maximum number of iterations is set to 50 and the receiver stops once a valid codeword is found. Further increasing the number of local iterations per overall iteration has little effect on BER performance. The quantization range is set to [−4, 4]. In Fig. 3, we first compare the performance of the symbol-based BCJR detector (curve labeled "No Quantization") with that of the bit-level BCJR detector (Curve labeled "Bit-Level BCJR, No Quantization"), where both receivers assume unquantized input signals. The symbol-based detector is shown to achieve a noticeable performance gain of 0.7 dB over the bit-level detector at the BER of 10-5 when the nonbinary LDPC code is used over the channel. Moreover, compared with a rate-0.917 (4656, 4268) binary LDPC code of the same code parameters (Curve labeled "Binary LDPC, No Quantization"), the nonbinary code performs better by 0.3 dB at the BER of 10-5 and it is also shown that a larger gain can be expected with increasingly higher SNR values. Due to its superior performance, it is meaningful to investigate the effect of quantization on the performance of the symbol-level BCJR detector when nonbinary LDPC codes are used as a channel coding scheme. Fig. 3 shows that with 4-bit uniform quantization, the proposed receiver based on the quantized symbol-level BCJR algorithm has a performance loss of 0.8 dB at the BER of 10-5 as compared with the receiver operating on unquantized signals. Increasing the number of quantization bits to n=5 reduces the gap to 0.3 dB; while the 6-bit uniform quantizer results in little performance degradation. For comparison purposes, we include in Fig. 3 the performance of the BCJR algorithm that directly uses the 4-bit quantized received signals in the conventional branch metric calculation (5) (Curve labeled "4-bit, Conventional").

We note that when severe constraints are placed on the number of quantization bits (e.g., n=4), the proposed receiver achieves a non-negligible performance gain of 0.2 dB at the BER of 10-5. Though not included in the figure, simulation results show that the proposed receiver suffers no performance degradation when the transition probability matrix computed at Eb/No=6.4 dB is used throughout the simulated SNR range.

In Fig. 3, it is also shown that we can achieve significantly better performance by using a non-uniform quantizer when the number of quantization bits is limited to be a small value. The minimum-mean-square-error (MMSE)-optimized non-uniform quantizer is designed based on the Lloyd algorithm [10]. Fig. 3 shows that the 4-bit non-uniform quantizer provides a noticeable performance gain of 0.6 dB over the uniform quantizer and operates close to the unquantized receiver within 0.3 dB at the BER of 10-5. An additional gain of 0.2 dB is obtained for 5-bit non-uniform quantization, which further reduces the performance loss to 0.1 dB at the BER of 10-5.

In Fig. 4, we present the average number of iterations required for various receivers. Compared with the receiver operating on unquantized signals (curve labeled "No Quantization"), the proposed receiver with n=5 non-uniform quantization and n=6 uniform quantization need a similar number of iterations to find a valid codeword at high SNRs, which is less than 5 for Eb/No⩾6.4 dB. This indicates that the complexity reduction can be achieved without significantly compromising convergence rates.

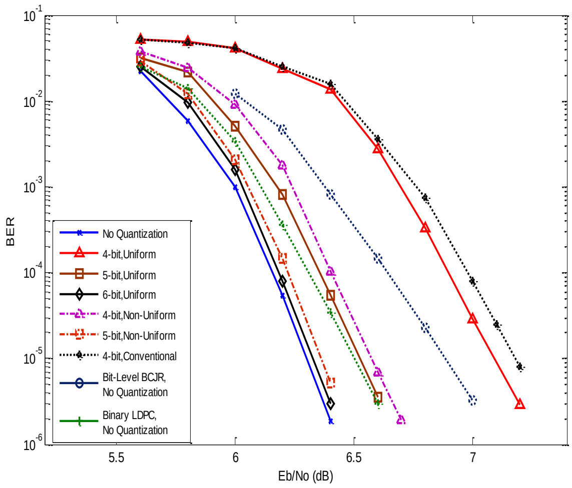

Fig. 3. BER performance of the proposed iterative receiver over a rate-0.917 (1164, 1067) nonbinary LDPC coded quantized ideal E2PR4 channel.

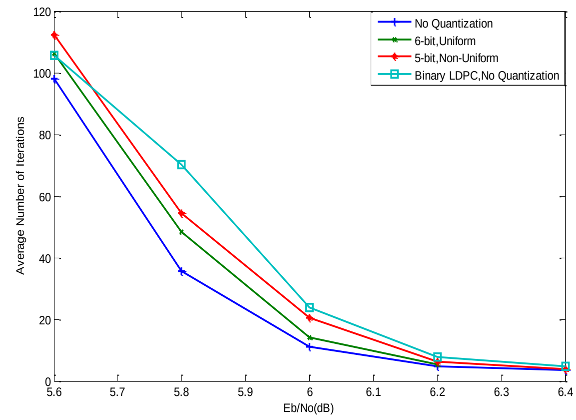

Fig. 4. Average number of iterations required for the proposed receiver over a rate-0.917 (1164, 1067) nonbinary LDPC coded quantized ideal E2PR4 channel.

B. BPMR Channels

In this section, we consider the performance of the proposed receiver over BPMR channels, which is a promising model for next-generation high-density magnetic recording systems [5]. The simulated BPMR channel corresponds to a recording density of 4 Tb/In^2 with bit-aspect-ratio (BAR) 1, square islands of side length 6.3 nm and height 8.7 nm, downtrack and crosstrack island period of 12.7 nm, respectively. The giant magnetoresistive (GMR) read-head has an element thickness 3nm and width 15nm, the downtrack and crosstrack sensor-to-shield gap 6nm and 9nm, respectively. We also assume an unshielded write head, island switching field distribution (SFD) 10%, island position jitter 10%, and island size variation 10, respectively. The 2-dimensional (2D) equalizer has a length of 11 and a width of 3. The equalizer is implemented with floating-point coefficients based on unquantized received signals. A length-4 generalized partial-response (GPR) target is designed by using the joint equalization and target optimization technique based on the monic-constrained MMSE criterion as described in [11]. It is also shown in [11] that the noise at the output of the equalizer is Gaussian distributed and its variance can be computed analytically. The details of the BPMR channel model and the equalizer design may be found in [5] and [11], respectively.

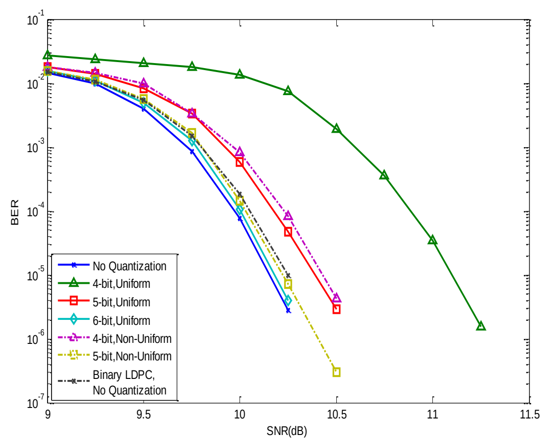

Fig. 5. BER performance of the proposed iterative receiver over a rate-0.938 (1552, 1455) nonbinary LDPC coded quantized BPMR channel.

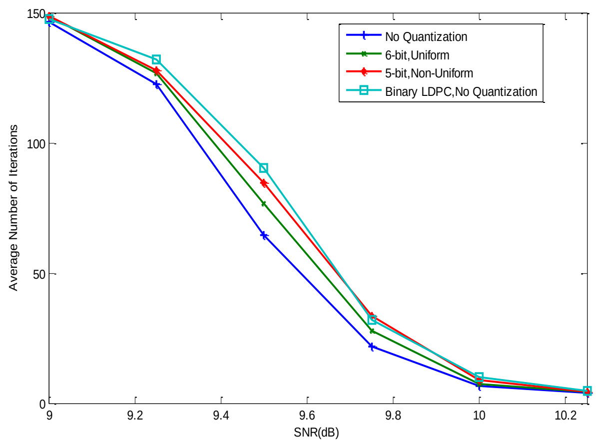

Fig. 6. Average number of iterations required for the proposed receiver over a rate-0.938 (1552, 1455) nonbinary LDPC coded quantized BPMR channel.

The LDPC code used in the simulation is a rate-0.938 (1552, 1455) nonbinary code over GF(23). For comparison, the performance of a binary (4556, 4365) LDPC code over the BPMR channel is also simulated. Fig. 5 shows that the nonbinary LDPC code with the symbol-level BCJR detection outperforms the binary LDPC code when unquantized channel observations are used as input signals. In addition, the 4-bit non-uniform quantization scheme provides a performance gain of 0.7 dB over the uniform quantization; while the 5-bit non-uniform quantization approaches the performance of unquantized channels within 0.1 dB at the BER of 10-5. The average number of iterations required by various receivers over BPMR channels is shown in Fig. 6, which shows that the proposed receiver needs more iterations to converge at low-to-medium SNR. However, all receivers have similar convergence rates when the SNR is increased to 10 dB, where only 2 iterations are needed to find a valid codeword.

V. Conclusion

In this paper, we consider the performance of turbo equalization over quantized nonbinary LDPC coded channels, where transition probabilities are computed in advance to reduce the computational complexity by avoiding floating-point multiplications in the symbol-level BCJR algorithm. Simulation results show that the proposed receiver approaches closely the performance of the conventional receiver operating on unquantized signals. Future work includes the performance evaluation of the Ungerboeck's metric [12] when applied to the BCJR algorithm, the pattern-dependent noise-predictive (PDNP) channel detector, as well as the fixed-point equalization/detection implementations.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Xiaowei Liu and Zhiliang Qin conducted conceptualization, methodology and writing-review. Nina Zhang and YingyingLi conducted the algorithm developments, validation and writing-original draft preparation in the paper. Luyan Xing and Qidong Lu conducted experimental data collection, literature and survey.

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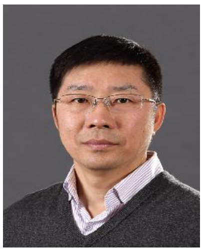

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